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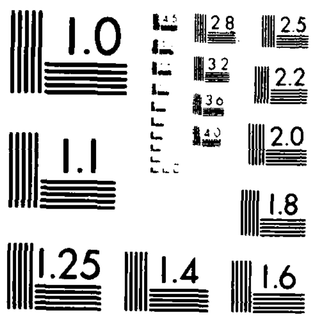


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# FOREIGN TECHNOLOGY DIVISION

THEORETICAL ANALYSIS OF LANGMUIR FLOW EFFECTS IN A RING LASER

by

Jiang Yanan



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THEORETICAL ANALYSIS OF LANGMUIR FLOW EFFECTS IN A RING LASER

Jiang Yanan (Department of Precision Instruments, Qinghua University)

ABSTRACT

Gas flow in a gain medium is one of the main sources of error in laser gyros. In this paper a quantitative discussion is given on shifts in frequency difference of oppositely directed travelling waves (ODTW) which is brought about by the flow of active atoms and non-active atoms in the gain medium.

Theoretical formulas and calculated values are given for Langmuir flows based on third-order theory. The focal point of this paper lies in the use of statistics to deal with shifts in frequency difference for Gaussian beams passing through the gain field and the flow velocity field. Also, calculation formulas have been given for the various cases.

I. FOREWORD

In a ring laser, travelling wave oscillations are maintained in both the clockwise (CW) and counterclockwise (CCW) directions of propagation. Since a direct current (dc) discharge is employed to maintain activation of the gaseous gain medium, gaseous flow occurring in the discharge tube cannot be avoided. The typical value of its flow rate in a low-power He-Ne ring laser is approximately a few mm/s (this kind of flow is called Langmuir flow). The result can be that splitting can occur in the frequencies of the CW and CCW travelling waves and its typical value $\Delta\nu_L$ is approximately several tens of Hz.

As we know, for a ring laser having rotation $\Omega$ with respect to inertial space, due to the relativistic effect, a frequency difference will occur between the CW and CCW travelling waves:

$$\Delta\nu_{\Omega} = |\nu^{CW} - \nu^{CCW}| = \frac{4S\cdot\Omega}{\lambda L} \qquad (1)$$

where S is the area enclosed by the ring cavity; L is the cavity length; $\lambda$ is the laser wavelength.

$K = \frac{4S}{\lambda L}$ is known as the scale factor of the gyro. In order to have the concept of fixed quantity, the typical value of K in a practical laser gyro is approximately 1Hz/degree/hour. Consequently, to achieve a sensing accuracy greater than $10^{-2}$ degrees/hour, various other sources of error which can give rise to a shift in frequency difference $\Delta\nu$ between the CW and CCW travelling waves must be stabilized below $10^{-2}$Hz.

Above we have already referred to the Langmuir flow typical value $\Delta\nu_L \sim$ several tens of Hz. We can see that this term gives rise to the seriousness of the error source of laser gyro null shift.

For this reason, in a practical laser gyro a two-cathode discharge system of operation is used (see Fig. 1).

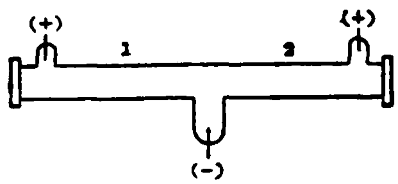


Fig. 1. Two-circuit discharge tubes 1, 2

Even employed in this manner, due to the complexity and instability of the mutual effects between Gaussian beams, Langmuir flows, and gain fields, it will still bring about noticeable normal value shifts and random shifts. Especially in four-frequency differential laser gyros, a number of primary error sources are eliminated as a result of the differential, but the shift induced by the Langmuir flow is doubled. Consequently, in high precision gyro devices, research into Langmuir flow effects is very important.

In He-Ne dc discharge gaseous gain tubes, the flow of so-called

gaseous atoms includes two types of flow, that of active atoms (Ne*) and non-active atoms (such as He and non-active Ne). Their directional flows can give rise to the above non-mutual effects - causing a frequency difference to occur between the CW and CCW travelling waves - i.e., a null shift. But the physical mechanisms of null shift which are brought about by the non-active atoms and the gain atoms are different and therefore must be discussed separately.

The Langmuir flow effects of non-active atoms are relatively simple. Based on the Fresnel pulling effect for the velocity of light in a moving medium, the term for shift in Langmuir flow for a non-active medium is

$$\Delta\nu'_L = -\frac{4S}{\lambda L}\left(1-\frac{1}{n^2}\right)\bar{\Omega}' \tag{2}$$

where $\bar{\Omega}'$ is the equivalent angular velocity of movement of the non-active medium relative to the ring cavity.

Calculations indicate that under normal conditions this term will be three orders of magnitude smaller than the Langmuir shift of the active medium. The main point of this paper lies in the discussion of the Langmuir flow effects of active atoms.

## II. BASIC EQUATIONS

See References [1~3]. The circular frequencies ($\omega_1$ and $\omega_2$) of the oppositely directed travelling waves satisfy the following equations

$$\omega_1 = \Omega_1 + \sigma_1 + \rho_1 I_1 + \tau_{12} I_2 \tag{3}$$

$$\omega_2 = \Omega_2 + \sigma_2 + \rho_2 I_2 + \tau_{21} I_1 \tag{4}$$

This is the result of the third-order theory of polarization intensity of the active medium. The corresponding term which pertains to lock-in has been omitted from the expressions because the problems discussed in this paper are not concerned with lock-in which means that the ring laser should operate at a point far removed from the lock-in region. With regard to the four-frequency device employed in this paper, this is completely satisfied. In the expressions, 1 and 2 represent CW and CCW travelling waves, respectively; $\Omega_{1,2}$

are the cavity resonance frequencies; $\sigma$ is the chromatic dispersion coefficient of the active medium; $\rho$ and $\tau$ are the gain saturation parameters of the medium.

The circular frequency difference between the CW and CCW travelling waves can be obtained from (3) and (4)

$$\omega = \Omega + (\sigma_2 - \sigma_1) + (\tau_{21} - \rho_1) I - (\tau_{12} - \rho_2) I_2 \tag{5}$$

where $\Omega = \Omega_2 - \Omega_1$ is the circular frequency difference of the cavity.

When we take into account radiation trapping effects and at the same time employ mean light intensity I and light intensity difference $i$

$$I = \frac{1}{2}(I_2 + I_1) \tag{6}$$

$$i = \frac{1}{2}(I_2 - I_1) \tag{7}$$

we can obtain

$$\begin{aligned}\omega = \Omega + (\sigma_2 - \sigma_1) + 2(\rho - \tau) i \\ - \Big[ \frac{2R}{G}(\sigma_2 \beta_2 - \sigma_1 \beta_1) \\ + (\tau_{12} - \tau_{21}) + (\rho_1 - \rho_2) \Big] I\end{aligned} \tag{8}$$

Omitting the term for light intensity difference, i.e., letting $i = 0$, and using the frequency parameter

$$\xi_i = \frac{\omega_i - \omega_0}{K u} \quad i = 1,\ 2$$

expression (8) can be expressed as

$$\omega = \Omega(1 + A) \tag{9}$$

$$A = \frac{G}{2Ku}\left(\frac{C}{L}\right)[S_\sigma + S_R + S_\tau + S_\rho] \tag{10}$$

Due to the presence of an active medium, A in expression (9) represents tuning for cavity frequency difference and is called the scale factor correction term. In expression (10), $S_\sigma$ $S_R$ $S_\tau$ and $S_\rho$ represent the difference correction terms for mode pulling, radiation trapping, mode self-repulsion, and mode mutual repulsion, respectively.

Let

$$S = S_\sigma + S_R + S_\tau + S_\rho \tag{11}$$

When the ratio for the two isotopes $Ne^{20}:Ne^{22} = 1:1$, these parameters are written as

$$S_r = \frac{\frac{1}{2}[Z'_r + \bar{Z}'_r]}{Z_{im}}$$

$$S_R = \frac{\left\{ -\frac{1}{2} RI[(Z'_r + \bar{Z}_r)(b+\bar{b}) + (Z_r + \bar{Z}_r)(b' + \bar{b}')] \right\}}{Z_{im}}$$

$$S_t = \frac{-\frac{1}{2}\eta I\left[\frac{Z'_i}{\xi} + \frac{\bar{Z}'_i}{(\xi - \xi_i)}\right]}{Z_{im}}$$

$$S_p = \frac{-\frac{1}{2}\eta I[Z''_i + \bar{Z}''_i]}{Z_{im}}$$

where $b = (Z_i - \eta Z'_r)/Z_{im}$, $Z_i$ and $Z_r$ represent the real component and the imaginary component of the plasma chromatic dispersion function; $\xi_i$ is the frequency interval of the two isotopes; $Z_{im} = Z_i(0)$ is the maximum value of this function. Calculations indicate that $S_r$ and $S_R$ will be one order of magnitude greater than $S_t$ and $S_p$ [3].

## III. GENERAL DESCRIPTION OF LANGMUIR FLOW EFFECTS

In the discharge tube of a ring laser, due to the dc discharge, the gain atoms have a certain velocity V. But due to the Doppler effect, the gain curves for travelling waves 1 and 2 give rise to 2KV circular frequency splitting. Consequently, the gyro frequency difference equation is changed to

$$\omega = \Omega(1 + A) - 2KV \cdot A \qquad (12)$$

This makes it clear that the term for circular frequency difference induced by Langmuir flow

$$\Delta\omega_L = -2KVA$$

This is induced by the splitting of the CW and CCW optical gain curves which leads to difference values occurring between the gain medium and the CW and CCW optical pulling and repulsion effects.

In a four-frequency differential ring laser, due to the simultaneous operation of the right- and left-hand-rotating gyros, in the differential output signal

$$\Delta\omega = \omega_R - \omega_L$$

for right- and left-hand rotation it will increase to

$$\Delta\omega = 4KV \cdot \bar{A} \tag{13}$$

This is the term for Langmuir flow effect circular frequency shift in a four-frequency differential laser gyro. In expression (13),

$$\bar{A} = \frac{1}{2}(A_{左} + A_{右}) \tag{14}$$

Figure 2 is a diagram of the operating principles of a four-frequency differential gyro device. The 90$^{o}$ quartz optical rotator causes the frequency interval between the right- and left-hand-rotating gyros to be $\frac{C}{2L}$ (half of the mode interval). The cavity controller and the piezoelectric element ensure that the right- and left-hand-rotating gyros operate at the symmetrical operation point of the gain curve. In order to avoid intermode competition and gain curve symmetry, the device is filled with equal proportions of $Ne^{20}$ and $Ne^{22}$. The magneto-optic Faraday element causes the right- and left-hand-rotating gyros to create equal and opposite off-frequency operating points in order to avoid the lock-in region.

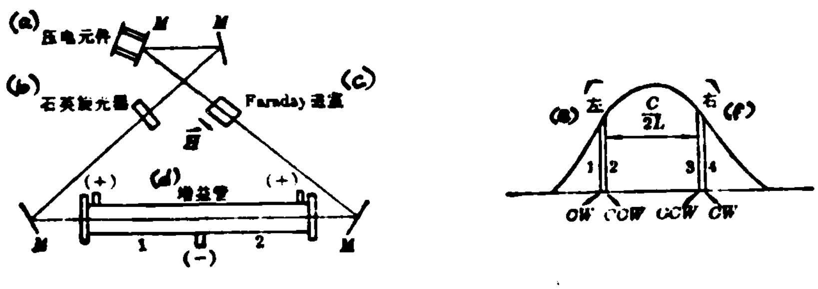


Fig. 2. Schematic of operating principles of a four-frequency gyro device (M: angled mirrors)

KEY: (a) piezoelectric element; (b) quartz optical rotator; (c) Faraday magnetic cell; (d) gain tube; (e) left; (f) right

The parameters of the four frequency device during these combined tests provide estimated values for a set of Langmuir flows.

Assuming the effective value of directional flow velocity of the gain atoms in the discharge tube is V = 1cm/s, then the frequency

splitting of the gain curve will be

$$\frac{KV}{\pi} = 0.0317\text{MHz},$$

and the Langmuir flow frequency shift induced thereby

$$\Delta\nu_L = \frac{4V}{\lambda}\bar{A} \tag{15}$$

$$\bar{A} = \frac{C}{2L}\frac{G}{Ku}\bar{S} \tag{16}$$

In this device L = 63.3cm, and

$$\frac{C}{2L} = 241\text{MHz};$$

the He-Ne is charged to P=3.3torr, and $\eta$ =0.2; bandwidth of the omitted light $\Delta\xi$ =0.4, and gain/loss ratio is $G/r$ =1.17; since $r$ =3%, G=3.51%. Hence

$$\frac{C}{2L}\frac{G}{Ku} = 1.34\times10^{-3},$$
$$\bar{A} = 0.963\times10^{-3},$$
$$\Delta\nu_L = 61.1\text{Hz}.$$

In the two-cathode discharge tube, the gain and the atom flow velocity for the two sections of the discharge tube were recorded as $G_1$, $G_2$, $V_1$, and $V_2$, respectively. Then

$$\Delta\nu_L = \frac{4(G_2V_2 - G_1V_1)}{\lambda G}\bar{A} \tag{17}$$

Assuming $G_1 = G_2 = \frac{1}{2}G$,

$$\Delta\nu_L = 2\frac{V_2 - V_1}{\lambda}\bar{A}.$$

When $V_2 - V_1$=1cm/s, $\Delta\nu_L \approx 30.6$ Hz.

When the voltage difference $\Delta I$ between the two sections of the discharge tube was 1mA, we obtained a test measurement of $\Delta\nu_L$ =52Hz. This indicates that the flow velocity differential between the two sections of the discharge tube at this time was

$$V_2 - V_1 = 1.7\text{cm/s}.$$

Above, beginning with directional flow velocity V of the gain atoms in the discharge tube, we have presented a general discussion on the origin of Langmuir flow shift and its numerical calculation.

$\Delta_{\nu_L}$ is directly proportional to G and V, but, generally speaking, in a gaseous discharge tube G and V are not constants at all. They are distributed radially and axially according to fixed rules. Therefore, in order to further the discussion of Langmuir flow null shift, below we analyze the field distribution of gain coefficients and flow velocities.

## IV. TRANSVERSE DISTRIBUTION OF VELOCITY AND GAIN COEFFICIENT FIELDS

### (1) Flow velocity field

In the cylindrical region of a closed dc He-Ne discharge tube there are two types of opposing flows of gaseous atoms. One type is Langmuir flow, and its direction is from the cathode to the anode. Its intrinsic quality is the negative load effect due to the discharge tube wall. When the ion free path $\lambda_+ \ll a$ (the radius of the discharge tube), the velocity distribution of $r$ along the radius of the capillary tube is

$$V_L(r) = V_L(0)\left[1 - e^{\frac{r-a}{\lambda_+}}\right] \quad (18)$$

[4]

In a discharge tube with 3torr and He:Ne=7:1, $\lambda_+ \sim 0.05$mm.

As a result of the Langmuir flow a pressure difference is created between the cathode and the anode of the closed discharge tube. Under the effects of the pressure difference a second type flow is also formed in the discharge tube: a viscid flow (Poiseuille flow) $V_p(r)$, and its distribution is

$$V_p(r) = V_p(0)\left[1 - \left(\frac{r}{a}\right)^2\right] \quad (19)$$

[4]

In the closed discharge tube a stationary flow forms under equilibrium conditions, consequently, we have

$$\int_0^{2\pi}\int_0^a V_p(r)\, r\, dr\, d\theta = \int_0^{2\pi}\int_0^a V_L(r)\, r\, dr\, d\theta$$

Assuming $a$ = 0.75mm, then $a/\lambda_+ \simeq 15$, and we have

$$V_L(0) \approx \frac{4}{7} V_p(0)$$

The flow velocity of the gas atoms $V(r)$ is a combination of the above two types of opposing flows, i.e.,

$$\begin{aligned} V(r) &= V_p(r) - V_L(r) \\ &= V_0\left[1 - \frac{7}{3}\left(\frac{r}{a}\right)^2 + \frac{4}{3} e^{\frac{r-a}{h}}\right] \end{aligned} \tag{20}$$

From expression (20) it is obvious that, at

$$\begin{cases} r = a \\ r = 0.65a \end{cases}, \quad V(r) = 0$$

In the region of $r < 0.6$, the above expression is simplified as

$$V(r) = V_0\left[1 - \frac{7}{8}\left(\frac{r}{a}\right)^2\right] \tag{21}$$

Expression (21) can be used for flow velocities in the region through which the Gaussian beams pass as discussed in this paper

Figure 3 presents a flow velocity diagram based on expression (20). It is obvious from the figure that in the central region of the discharge tube the atoms flow from the cathode to the anode. But in the fringe region of the discharge tube they flow from the anode to the cathode. On the cross section the total flow rate toward the cathode and toward the anode are equal [5].

But it should be pointed out that the flow velocity field, in reality, is more complicated than this and requires further research.

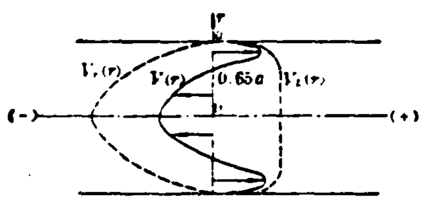


Fig. 3. The transverse distribution of flow velocities inside the discharge tube $V(r)$

(2) Gain coefficient field

The illustration proves that the distribution of gain coefficients (Fig. 4), determined by the distribution of electron densities $n_e$ in the discharge tube, is

$$G(r) = G_0 J_0\left(\frac{2.4r}{a}\right)$$

But the generally defined simplex gain coefficient will be the mean value of $G(r)$ within the region through which the Gaussian beams pass.

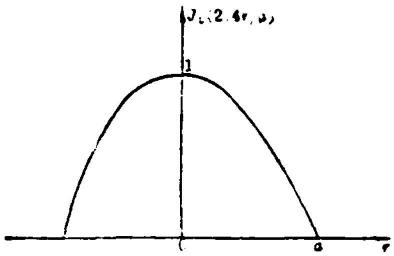


Fig. 4. The transverse distribution of the gain field within the discharge tube $G(r) = G_0 J_0(r)$

V. LANGMUIR FLOW NULL SHIFT FOR THE CASE WHERE THE GAUSSIAN BEAMS ARE ECCENTRIC

The above discussed gain coefficients $G(r)$ and flow velocities $V(r)$ are transversely distributed across the discharge tube. The problem we faced in this work was Gaussian beams with an intensity distribution of $(e^{-r^2/\omega^2})$ passing through the gain field and flow velocity field in the discharge tube at an offset of $a$ which produced a Langmuir flow null shift $\Delta\nu_L$.

Since G and V are functions of $r$, then $\Delta\nu_L$ is also certainly a function of $r$. Therefore, the Langmuir null shift which played a part in the experiments was directly proportional to the statistical mean value of $G(r)\cdot V(r)$ where $e^{-r^2/\omega^2}$ was the weight factor.

From expressions (15) and (16)

$$\Delta\nu_L = QGV \tag{23}$$

where

$$Q = \bar{A}\frac{4}{\lambda G} \tag{24}$$

Since there is an offset value $a$ between the Gaussian beams and the G and V fields, we used cylindrical coordinates $(r, \theta)$ whose origin O coincides with the center of the Gaussian beams. Consequently, in these coordinates the gain field and flow velocity field with

the discharge tube center O' as their symmetrical center were $G(r)$ and $V(r)$, respectively, and $\overline{OO'} = a$, see Figure 5.

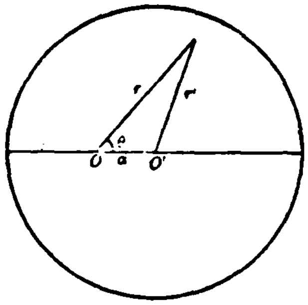


Fig. 5. The center of the Gaussian beams is offset from the center of the discharge tube o', the amount of offset is $a$

$$(r')^2 = r^2 + a^2 - 2ra\cos\theta$$

$$\langle \Delta\nu_L \rangle_a = Q\langle G \cdot V \rangle_a$$

$$= Q\frac{\int_0^{2\pi}\int_0^{\omega_0} e^{-r^2/\omega^2} G_0 J_0(2.4r'/a) V(r') r\,dr\,d\theta}{\int_0^{2\pi}\int_0^{\omega_0} e^{-r^2/\omega^2} r\,dr\,d\theta} \tag{25}$$

By calculation we obtain

$$\begin{aligned}\langle \Delta\nu_L \rangle_a &= QG_0V_0\Big[0.6951 - 2.735\Big(\frac{a}{a}\Big)^2 \\ &\quad + 3.879\Big(\frac{a}{a}\Big)^4\Big] \\ &= QG_0V_0 f(a)\end{aligned} \tag{26}$$

After retaining the second term

$$f(a) = 0.6951 - 2.735\Big(\frac{a}{a}\Big)^2 \tag{27}$$

In a closed two-cathode discharge tube

$$\begin{aligned}\langle \Delta\nu_L \rangle_a = Q[&G_{10}V_{10}f(a_1) \\ &- G_{20}V_{20}f(a_2)]\end{aligned} \tag{28}$$

where $G_{10}$, $G_{20}$, $V_{10}$, and $V_{20}$ are the gain coefficients and flow velocities at the centers of the two sections 1 and 2 of the discharge tube, respectively, see Figure 2.

$G_0 = G_{10} + G_{20}$, and assuming the two sections of the discharge tube are symmetrically centered, i.e., $a_1 = a_2 = a$, then

$$\langle \Delta\nu_L \rangle_a = Q[G_{10}V_{10} - G_{20}V_{20}]f(a) \tag{29}$$

It is obvious that when $G_{10}V_{10}=G_{20}V_{20}$, $\langle \Delta\nu_L \rangle_a = 0$, i.e., Langmuir flow null shift is zero. But when

$$G_{10} = G_{20} = \frac{G_0}{2},$$

$$\langle \Delta\nu_L \rangle_a = QG_0\left(\frac{V_{10}-V_{20}}{2}\right)f(a) \tag{30}$$

VI. RELATIVE SYSTEMATIC NULL SHIFT $\gamma_a$ AND RANDOM NULL SHIFT $\beta_a$

What this work is concerned with is that since initial tuning is poor, the basic mode of oscillation in the ring cavity has an eccentricity $a$ with respect to the discharge tube. Thus, the additional null shift which is induced as well as factors resulting from the instabilities of the cavity during the experiments, cause $a$ to be in continuous change $(\Delta a)$ and causes random changes in induced null shift.

To describe these relative changes in null shift, we introduce the two parameters $\gamma_a$ and $\beta_a$. They are defined as

$$\gamma_a \equiv \frac{\langle \Delta\nu_L \rangle_a - \langle \Delta\nu_L \rangle_{a=0}}{\langle \Delta\nu_L \rangle_{a=0}} = -3.935\left(\frac{a}{a}\right)^2 \tag{31}$$

$$\beta_a \equiv \frac{\dfrac{d\langle \Delta\nu_L \rangle_{a-\Delta a/2}}{da} \cdot \Delta a}{\langle \Delta\nu_L \rangle_0} = -\frac{3.935(2a+\Delta a)\Delta a}{a^2} \tag{32}$$

If $a$ = 0.75mm, see Table 1.

Table 1. The amount of relative change $\gamma_a$ in Langmuir flow null shift when the Gaussian beam offset from the center of the gain tube is $a$

| $a$ (毫米) (a) | $\gamma_a$ |
|---|---|
| 0.01 | $-7.0\times10^{-4}$ |
| 0.075 | $-3.9\times10^{-2}$ |
| 0.1 | $-7.0\times10^{-2}$ |
| 0.2 | $-0.25$ |

KEY: (a) mm

If $a$ = 0.75mm, $a$ = 0.1mm, see Table 2

It is obvious from the above that tuning of the cavity will result in offset $a$ being as small as possible and thus will cause the random changes in Langmuir flow null shift to be relatively small under the effects of an equivalent random offset $\Delta a$.

Table 2. At a Gaussian beam offset of $a$ = 0.1mm from the center of the gain tube, the amount of relative change in Langmuir flow null shift $\beta_a$ which is induced as a result of random changes in $a$, i.e.,

| $\Delta a$ [illegible] (a) | $\beta_a$ |
|---|---|
| 0.001 | $-1.4\times10^{-3}$ |
| 0.01 | $-1.47\times10^{-2}$ |
| 0.02 | $-3.08\times10^{-2}$ |
| 0.03 | $-4.83\times10^{-2}$ |
| 0.05 | $-8.75\times10^{-2}$ |

KEY: (a) mm

## VII. LANGMUIR FLOW NULL SHIFT IN THE CASE WHERE GAUSSIAN BEAMS PASS THROUGH OBLIQUELY[7]

In an actual ring cavity there are cases where the Gaussian beams pass obliquely through the discharge tube. In such cases, with regard to the central z-axis of the oblique Gaussian beams, the distribution of G and V differs at different z, and this corresponds to the different amounts of offset $a(z)$ at different z, see Figure 6.

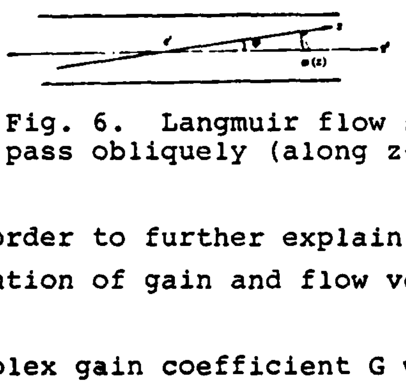

Fig. 6. Langmuir flow shift when the Gaussian beams pass obliquely (along z-axis) through the gain tube

In order to further explain this problem we must discuss the distribution of gain and flow velocity along the z-axis.

Simplex gain coefficient G was used in our earlier discussion which inevitably overlooked gain distribution along the z-axis. Here we introduce gain density $g$, i.e., the concept of gain value per unit length. In cylindrical coordinates this becomes $g(r, \theta, z)$, and

$$G(r\theta) = \int_0^{l_0} g(r, \theta, z) dz$$

The relationship between $g(r, \theta, z)$ and $\overline{G}$ is

$$\overline{G} = \frac{\int_0^{l_0}\int_0^{2\pi}\int_0^{\omega_0} g(r, \theta, z) r\, dr\, d\theta\, dz}{\int_0^{2\pi}\int_0^{\omega_0} r\, dr\, d\theta} \tag{33}$$

$0 \to l_0$ is the length of the gain region.

The above expression makes it clear that simplex gain $\overline{G}$ is the radial mean and axial cummulative value of gain density $g$ within the effective region of the Gaussian beams. It is obvious that as the Gaussian beams travel along the center of the discharge tube the simplex gain $\overline{G}$ supplied to it by the gain tube will also change.

For Langmuir flow null shift $\langle \Delta\nu_L \rangle_{a(z)}$ when the Gaussian beams pass obliquely through the discharge tube, $a(z)$ is offset of the center of the Gaussian beams, at z, from the center of the tube.

$$\langle \Delta\nu_L \rangle_{a(z)} = \frac{\left\{ Q\int_0^{l_0}\left[\int_0^{2\pi}\int_0^{\omega_0} e^{-r/\omega_0} g(r', z) \times V(r', z) r\, dr\, d\theta\right] dz \right\}}{\int_0^{2\pi}\int_0^{\omega_0} e^{-r/\omega_0} r\, dr\, d\theta}$$
$$= Q\int_0^{l_0} g_0 V_0 f[a(z)] dz \tag{34}$$

In a two-cathode discharge tube, assuming the length of the two sections of the discharge tube are the same, we have

$$\langle \Delta\nu_L \rangle_{a(z)} = Q\left\{ G_{10} V_{10} \frac{\int_0^{\frac{l_0}{2}} f[a_1(z)] dz}{\frac{l_0}{2}} - G_{20} V_{20} \frac{\int_{\frac{l_0}{2}}^{l_0} f[a_2(z)] dz}{\frac{l_0}{2}} \right\} \tag{35}$$

where $G_{10} = g_{10}\frac{l_0}{2}$, $G_{20} = g_{20}\frac{l_0}{2}$

The following goes a step further in giving the actual form of $\alpha(z)$, by performing integration. Assume that the Gaussian beam center z-axis and the discharge tube center z'-axis spatially intersect, the intersection point is at $l'$, and both intersection angles are $\varphi$. Then $\alpha_1(z)$ and $\alpha_2(z)$ can use the unified expression

$$\alpha(z) = (z - l')\tan\varphi \tag{36}$$

Since $\alpha(z)$ uses $l'$ and $\varphi$ as parameters it will be more precise to rewrite the above $\langle\Delta\nu_L\rangle_{\alpha(z)}$ as $\langle\Delta\nu_L\rangle_{l',\varphi}$

Furthermore, assuming $G_{20} = G_{10} = \frac{1}{2} G_0$ , $V_{10} = V_{20} = V_0$ and we then have

$$\langle\Delta\nu_L\rangle_{l',\varphi} = \frac{2.735 \times Q\, G_0 V_0 l_0^2}{a^2} \times \left(\frac{1}{2} - \frac{l'}{l_0}\right)\tan^2\varphi \tag{37}$$

The amount of relative change $\beta_\varphi$ and $\beta_{l'}$ in $\langle\Delta\nu_L\rangle_{l',\varphi}$, induced when $\varphi$ or $l'$ change (due to deformation of the cavity), is defined as

$$\beta_\varphi \equiv \frac{\frac{d\langle\Delta\nu_L\rangle_{l',\varphi}}{d\varphi}\cdot\Delta\varphi}{\langle\Delta\nu_L\rangle_0} \tag{38}$$

by calculation we obtain

$$\beta_\varphi = 7.86\left(\frac{l_0}{a}\right)^2\left(\frac{1}{2} - \frac{l'}{l_0}\right)\tan\varphi\cdot\Delta\varphi \tag{39}$$

$$\beta_{l'} \equiv \frac{\frac{d\langle\Delta\nu_L\rangle_{l',\varphi}}{dl'}\cdot\Delta l'}{\langle\Delta\nu_L\rangle_0} \tag{40}$$

by calculation we obtain

$$\beta_{l'} = -3.93(\tan^2\varphi)\cdot\frac{l_0}{a^2}\Delta l' \tag{41}$$

If the discharge tube gain region length $l_0$ = 200mm, $a$ = 0.75mm, we then have

a) $\beta_\varphi$ , assuming $l' = \frac{l}{4}$ , $l_0 = 2'$, is listed in Table 3.

Table 3. For Gaussian beams passing obliquely through the gain tube, the amount of relative change $\beta_\varphi$ in Langmuir flow null shift which is induced as a result of changes in oblique angle $\varphi$

| $\Delta\varphi$ | $\beta_\varphi$ |
|---|---|
| 10'' | $8.5\times10^{-3}$ |
| 1' | $2.1\times10^{-2}$ |
| 2' | $3.5\times10^{-2}$ |

b) $\beta_{l'}$, assuming $\varphi$ = 2', is listed in Table 4.

Table 4. For Gaussian beams passing obliquely through the gain tube, the amount of relative change $\beta_{l'}$ in Langmuir flow null shift which is induced as a result of changes in intersection point $l'$

| $l'$ [illegible] | $\beta_{l'}$ |
|---|---|
| 1 | $-3.5\times10^{-4}$ |
| 10 | $-3.5\times10^{-3}$ |

The above gives us $\beta_a$, $\beta_{l'}$, and $\beta_\varphi$ which represent the amounts of relative change in Langmuir flow null shift which is induced when the Gaussian beam center z-axis has a translation motion or oblique angle with respect to the discharge tube center z'-axis. However, under normal conditions, z and z' do not spatially intersect. It can be proved that the shortest distance between z and z' will be perpendicular to z and z'. After z translates for this shortest distance along this perpendicular line, it will intersect the z'-axis at $l'$ with an intersection angle of $\varphi$. Then we have

$$a^2(z) = a_0^2 + (z - l')^2 \tan^2\varphi$$

$$f(a) = 0.6951 - \frac{2.735}{a^2}[a_0^2 - (z - l')^2 \tan^2\varphi]$$

It is obvious that under these conditions

$$\beta = \beta_{a_0} + \beta_{l'} + \beta_\varphi \tag{42}$$

where $\beta_{a_0}$, $\beta_{l'}$, and $\beta_\varphi$ are based on expressions (32), (41), and (39), respectively. We will not indulge in further discussion at this time.

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